

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

線性代數 (二)

MATH 104A / GEAI 1209A: Linear Algebra II

第二次期中考

April 29, 2024

Midterm 2

姓名 Name : _____

學號 Student ID # : _____

Lecturer: Jephian Lin 林晉宏

Contents: cover page,
5 pages of questions,
score page at the end

To be answered: on the test paper

Duration: **110 minutes**

Total points: **20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear function and $\beta = \{\mathbf{u}_1, \mathbf{u}_2\}$ a basis of \mathbb{R}^2 such that

$$[f]_{\beta}^{\beta} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}, \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \text{ and } \mathbf{u}_2 = \begin{bmatrix} 6 \\ 7 \end{bmatrix}.$$

Let \mathcal{E}_2 be the standard basis of \mathbb{R}^2 .

(a) [1pt] Find $f(\mathbf{u}_2)$.

(b) [1pt] Find $f(\mathbf{u}_1 + \mathbf{u}_2)$.

(c) [1pt] Find $[\text{id}]_{\beta}^{\mathcal{E}_2}$.

(d) [1pt] Find $[\text{id}]_{\mathcal{E}_2}^{\beta}$.

(e) [1pt] Find $f\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix}\right)$.

2. [3pt] Let

$$A = \begin{bmatrix} 3 & 1 & 1 & 1 & 3 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 1 & 3 \end{bmatrix}.$$

Find the characteristic polynomial of A .

3. [2pt] Let

$$A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}.$$

Diagonalize A or provide reasons showing A is not diagonalizable.

4. Let

$$A = \begin{bmatrix} 6 & -8 & -4 \\ -1 & 1 & 1 \\ 10 & -14 & -8 \end{bmatrix}.$$

(a) [2pt] Find the characteristic polynomial and all eigenvalues of A .

(b) [3pt] Find a basis β such that $[f_A]_{\beta}^{\beta}$ is diagonal.

5. [5pt] Mathematical essay: Write a few paragraphs to introduce *diagonalization*.

Your score will be based on the following criteria.

- The definition is clear.
- Some sentences are added to explain the definition.
- Examples or pictures are included to help understanding.
- The sentences are complete.

6. [extra 2pt] Solve a_n in the recurrence relation

$$\begin{cases} a_{n+2} = a_{n+1} + 2a_n, \\ a_0 = 0, a_1 = 1. \end{cases}$$

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	