國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

**MATH107A / GEAI1216A**

**數學導論（一）** **Introduction to Mathematics I**

**第二次期中考** November 28, 2025 **Midterm 2**

姓名 Name : _____

學號 Student ID # : _____

| | |
|---|---|
| Lecturer: | Jephian Lin 林晉宏 |
| Contents: | cover page, |
| | **5 pages** of questions, |
| | score page at the end |
| To be answered: | on the test paper |
| Duration: | **110 minutes** |
| Total points: | **20 points** + 2 extra points |

**Do not open this packet until instructed to do so.**

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. [1pt] Give an example of "*d divides n*".

2. [2pt] Prove that the sum of two even numbers is even, while the sum of two odd numbers is even.

3. [2pt] Prove that
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$
for any positive integer $n$.

4. [1pt] Let $X = \{1, 2, 3\}$ and $Y = \{a, b\}$. List all elements in the Cartesian product $X \times Y$.

5. [2pt] Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

6. [2pt] Prove or disprove: $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$. Here $\mathcal{P}(S)$ stands for the power set of $S$.

7. [1pt] Let $X = \{1, 2, 3\}$ and $Y = \{a, b\}$. Consider the function $f : X \to Y$ defined by $f(1) = f(2) = f(3) = a$. What are the domain, the codomain, the graph, and the range of $f$?

8. [2pt] Let $\mathbb{R}_+$ be the set of all positive real numbers. Define $g : \mathbb{R}_+ \to \mathbb{R}_+$ by $g(x) = x^2$. Prove or disprove that $g$ is injective (one-to-one).

9. [2pt] For the same function $g$, prove or disprove that $g$ is surjective (onto).

10. [1pt] Consder the set

$$Q = \{(p, q) : p, q \in \mathbb{Z} \text{ and } p, q > 0\}.$$

Define

$$E = \{\langle (p_1, q_1), (p_2, q_2)\rangle \in Q^2 : p_1 q_2 = p_2 q_1\}.$$

Find two elements from $E$.

11. [2pt] For the same set $E$, show that $E$ is an equivalence.

12. [2pt] Show that the relation "divides" on $\mathbb{N} \setminus \{0\}$ is a partial order.

13. [extra 2pt] Find $a, b \in \mathbb{Z}$ such that $31a + 53b = 1$.

**[END]**

| Page | Points | Score |
|---|---|---|
| 1 | 5 | |
| 2 | 5 | |
| 3 | 5 | |
| 4 | 5 | |
| 5 | 2 | |
| Total | 20 (+2) | |