國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（二） **MATH 104A / GEAI 1209A: Linear Algebra II**

期末考 June 4, 2025 **Final Exam**

姓名 Name : _____

學號 Student ID # : _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **6 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 7 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. Let
$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}, C = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}, \text{ and } D = \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix}.$$

   (a) [1pt] State the definition of when two matrices are similar.

   (b) [1pt] Is A similar to B? **Provide your reasons.**

   (c) [1pt] Is B similar to C? **Provide your reasons.**

   (d) [1pt] Is B similar to D? **Provide your reasons.**

   (e) [1pt] Find a matrix M such that M is a similar to B and every entry of M is nonzero. **Justify your answer.**

2. [5pt] Let

$$A = \begin{bmatrix} 0 & 0 & 30 \\ 1 & 0 & -31 \\ 0 & 1 & 10 \end{bmatrix}.$$

Find a diagonal matrix D and an invertible matrix Q such that $D = Q^{-1}AQ$.

3. [5pt] Solve the recurrence relation

$$a_{n+2} = a_{n+1} + 6a_n,$$
$$a_0 = 0,\ a_1 = 1$$

and find a general form for a_n, in terms of n.

4. [5pt] Mathematical essay: Write a few paragraphs to introduce the *characteristic polynomial*.

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

5. [extra 5pt] Let

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 8 & -12 & 6 \end{bmatrix} \text{ and } J = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

Find an invertible matrix Q such that $J = Q^{-1}AQ$.

6. [extra 2pt] Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find an invertibile matrix Q such that $D = Q^{-1}AQ$.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	2	
Total	20 (+7)	