國立中山大學   NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（二）   **MATH 104A / GEAI 1209A: Linear Algebra II**

第一次期中考   March 19, 2025   **Midterm 1**

姓名 Name   : _____

學號 Student ID #   : _____

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page,
	5 pages of questions,
	score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** $+ 2$ extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.

- Using the calculator is not allowed (and not necessary) for this exam.

- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.

- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.

- Please answer the problems in English.

1. Let A be a 3×3 matrix whose rows are $\mathbf{x}, \mathbf{y}, \mathbf{z}$. Suppose $\det(A) = 3$.

   (a) [1pt] Let B be the 3×3 matrix whose rows are $\mathbf{x}, 3\mathbf{x} + \mathbf{y}, \mathbf{z}$. Find $\det(B)$.

   (b) [1pt] Let B be the 3×3 matrix whose rows are $\mathbf{x}, \mathbf{y}, 5\mathbf{z}$. Find $\det(B)$.

   (c) [1pt] Let B be the 3×3 matrix whose rows are $\mathbf{z}, \mathbf{y}, \mathbf{x}$. Find $\det(B)$.

   (d) [1pt] Let B be the 3×3 matrix whose rows are $\mathbf{y}, \mathbf{z}, \mathbf{x}$. Find $\det(B)$.

   (e) [1pt] Let B be the 3×3 matrix whose rows are $\mathbf{x} + \mathbf{y}, \mathbf{y} + \mathbf{z}, \mathbf{z} + \mathbf{x}$. Find $\det(B)$.

2. Find the determinant of

$$
A = \begin{bmatrix}
1 & -1 & 2 & 2 & -2 \\
-2 & 3 & -6 & -4 & 5 \\
-7 & 9 & -17 & -13 & 14 \\
-15 & 24 & -48 & -30 & 42 \\
14 & -17 & 32 & 27 & -26
\end{bmatrix}.
$$

3. [2pt] State the definition of the determinant of a matrix, using the row operations.

4. [1pt] Use the definition to prove that if A is a square matrix whose 5-th row is 10 times its 4-th row, then $\det(A) = 0$.

5. [2pt] Use the definition to prove that if the rows of A are dependent, then $\det(A) = 0$.

6. [5pt] Mathematical essay: Write a few paragraphs to introduce the *permutation expansion.*

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

7. [extra 2pt] Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Find $\det(A)$.

[**END**]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	