國立中山大學 NATIONAL SUN YAT-SEN UNIVERSITY

線性代數（二） **MATH 104A / GEAI 1209A: Linear Algebra II**

第一次期中考 March 19, 2025 **Midterm 1**

姓名 Name : solution .

學號 Student ID # : __________

Lecturer:	Jephian Lin 林晉宏
Contents:	cover page, **5 pages** of questions, score page at the end
To be answered:	on the test paper
Duration:	**110 minutes**
Total points:	**20 points** + 2 extra points

Do not open this packet until instructed to do so.

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. Let A be a 3×3 matrix whose rows are $\mathbf{x}, \mathbf{y}, \mathbf{z}$. Suppose $\det(A) = 5$.

(a) [1pt] Let B be the 3×3 matrix whose rows are $\mathbf{x}, 3\mathbf{x} + \mathbf{y}, \mathbf{z}$. Find $\det(B)$.

$A \xrightarrow{3\rho_1 + \rho_2} B$, so $\det(B) = \det(A) = \underline{\underline{5}}$

(b) [1pt] Let B be the 3×3 matrix whose rows are $\mathbf{x}, \mathbf{y}, 5\mathbf{z}$. Find $\det(B)$.

$A \xrightarrow{5\rho_3} B$, so $\det(B) = 5\det(A) = \underline{\underline{25}}$

(c) [1pt] Let B be the 3×3 matrix whose rows are $\mathbf{z}, \mathbf{y}, \mathbf{x}$. Find $\det(B)$.

$A \xrightarrow{\rho_1 \leftrightarrow \rho_3} B$, so $\det(B) = -\det(A) = \underline{\underline{-5}}$

(d) [1pt] Let B be the 3×3 matrix whose rows are $\mathbf{y}, \mathbf{z}, \mathbf{x}$. Find $\det(B)$.

$A \xrightarrow[\rho_2 \leftrightarrow \rho_1]{\rho_3 \leftrightarrow \rho_2} B$, so $\det(B) = (-1)^2 \det(A) = \underline{\underline{5}}$

(e) [1pt] Let B be the 3×3 matrix whose rows are $\mathbf{x} + \mathbf{y}, \mathbf{y} + \mathbf{z}, \mathbf{z} + \mathbf{x}$. Find $\det(B)$.

~~A → B~~

$$\begin{vmatrix} - & x+y & - \\ - & y+z & - \\ - & z+x & - \end{vmatrix} \underset{\substack{\rho_2+\rho_1 \\ \rho_3+\rho_1}}{=} \begin{vmatrix} - & 2x+2y+2z & - \\ - & y+z & - \\ - & z+x & - \end{vmatrix} = 2 \begin{vmatrix} x+y+z \\ y+z \\ z+x \end{vmatrix}$$

$$\underset{\substack{-\rho_1+\rho_2 \\ -\rho_1+\rho_3}}{=} 2 \begin{vmatrix} x+y+z \\ -x \\ -y \end{vmatrix} = 2 \begin{vmatrix} z \\ x \\ y \end{vmatrix}$$

$$= \underline{\underline{10}}$$

2. Find the determinant of

$$A = \begin{bmatrix} 1 & 2 & 0 & -3 & 2 \\ -3 & -5 & -2 & 12 & -7 \\ -3 & -6 & 1 & 9 & -9 \\ 16 & 26 & 14 & -68 & 30 \\ -9 & -16 & -3 & 34 & -21 \end{bmatrix}.$$

$$|A| \xlongequal[\text{comb's}]{\text{row}} \begin{vmatrix} 1 & 2 & 0 & -3 & 2 \\ 0 & 1 & -2 & 3 & -1 \\ 0 & 0 & 1 & 0 & -3 \\ 0 & -6 & 14 & -20 & -2 \\ 0 & 2 & -3 & 7 & -3 \end{vmatrix} \xlongequal[\text{comb's}]{\text{row}} \begin{vmatrix} 1 & 2 & 0 & -3 & 2 \\ & 1 & -2 & 3 & -1 \\ & 0 & 1 & 0 & -3 \\ & 0 & 2 & -2 & -8 \\ & 0 & 1 & 1 & -1 \end{vmatrix}$$

$$\xlongequal[\text{comb's}]{\text{row}} \begin{vmatrix} 1 & 2 & 0 & -3 & 2 \\ & 1 & -2 & 3 & -1 \\ & & 1 & 0 & -3 \\ & & 0 & -2 & -2 \\ & & 0 & 1 & 2 \end{vmatrix} \xlongequal[\text{comb's}]{\text{row}} \begin{vmatrix} 1 & & & & \\ & 1 & & \sim & \\ & & 1 & & \\ & & & 0 & 2 \\ & & & 1 & 2 \end{vmatrix}$$

$$= - \begin{vmatrix} 1 & & & & \\ & 1 & & \sim & \\ & & 1 & & \\ & & & 1 & \\ & & & & 2 \end{vmatrix} = \underline{-2}.$$

3. [2pt] State the definition of the determinant of a matrix, using the row operations.

4. [1pt] Use the definition to prove that if A is a square matrix whose 5-th row is 10 times its 4-th row, then $\det(A) = 0$.

5. [2pt] Use the definition to prove that if the rows of A are dependent, then $\det(A) = 0$.

See ver. A :

6. [5pt] Mathematical essay: Write a few paragraphs to introduce the *permutation expansion.*

   Your score will be based on the following criteria.

   - The definition is clear.
   - Some sentences are added to explain the definition.
   - Examples or pictures are included to help understanding.
   - The sentences are complete.

7. [extra 2pt] Let

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Find $\det(A)$.

See ver. A.

[END]

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	