

國立中山大學

NATIONAL SUN YAT-SEN UNIVERSITY

線性代數 (二)

MATH 104A / GEAI 1209A: Linear Algebra II

第二次期中考

April 23, 2025

Midterm 2

姓名 Name : \_\_\_\_\_

學號 Student ID # : \_\_\_\_\_

Lecturer: Jephian Lin 林晉宏

Contents: cover page,  
**5 pages** of questions,  
score page at the end

To be answered: on the test paper

Duration: **110 minutes**

Total points: **20 points** + 2 extra points

**Do not open this packet until instructed to do so.**

Instructions:

- Enter your **Name** and **Student ID #** before you start.
- Using the calculator is not allowed (and not necessary) for this exam.
- Any work necessary to arrive at an answer must be shown on the examination paper. Marks will not be given for final answers that are not supported by appropriate work.
- Clearly indicate your final answer to each question either by **underlining it or circling it**. If multiple answers are shown then no marks will be awarded.
- Please answer the problems in English.

1. Let  $V$  be a vector space with a basis  $\mathcal{B} = \langle \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \rangle$ . Let  $W$  be a vector space with a basis  $\mathcal{D} = \langle \mathbf{y}_1, \mathbf{y}_2 \rangle$ . Suppose  $f : V \rightarrow W$  is a homomorphism such that

$$\text{Rep}_{\mathcal{B}, \mathcal{D}}(f) = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 0 \end{bmatrix}.$$

(a) [1pt] Find  $f(\mathbf{x}_1)$ .

(b) [1pt] Find  $f(\mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3)$ .

(c) [1pt] Find a vector  $\mathbf{x}$  in  $V$  such that  $f(\mathbf{x}) = 10\mathbf{y}_1 + 10\mathbf{y}_2$ .

(d) [1pt] Find  $\text{null}(f)$ .

(e) [1pt] Find  $\text{rank}(f)$ .

2. Let  $\mathcal{P}_3$  be the vector space of polynomials of degree at most 3 and  $\mathcal{B} = \langle 1, x, x^2, x^3 \rangle$  be a basis of  $\mathcal{P}_3$ . Let  $f : \mathcal{P}_3 \rightarrow \mathcal{P}_3$  be a homomorphism defined by  $p(x) \mapsto p'(x) - xp''(x)$ .

(a) [1pt] Find  $f(4 + 3x + 2x^2 + x^3)$ .

(b) [2pt] Find the matrix  $A = \text{Rep}_{\mathcal{B}, \mathcal{B}}(f)$  such that  $A \text{Rep}_{\mathcal{B}}(p(x)) = \text{Rep}_{\mathcal{B}} f(p(x))$  for all  $p(x) \in \mathcal{P}_3$ .

(c) [2pt] Find a polynomial  $p(x)$  such that  $f(p(x)) = 1 + x^2$ ,  $p(0) = 10$ ,  $p(3) = 4$ .

3. [2pt] State the definition of an isomorphism.

4. [3pt] Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a function defined by

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} x + 2y \\ x + 3y \end{bmatrix}.$$

Show that  $f$  is an isomorphism.

5. [5pt] Mathematical essay: Write a few paragraphs to introduce a *homomorphism*.

Your score will be based on the following criteria.

- The definition is clear.
- Some sentences are added to explain the definition.
- Examples or pictures are included to help understanding.
- The sentences are complete.

6. [extra 2pt] Define the plane

$$P = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + 2y + 3z \right\}.$$

Find a matrix  $A$  such that  $A\mathbf{v}$  is the projection of  $\mathbf{v}$  onto the plane  $P$  for any  $\mathbf{v} \in \mathbb{R}^3$ .

**[END]**

Page	Points	Score
1	5	
2	5	
3	5	
4	5	
5	2	
Total	20 (+2)	