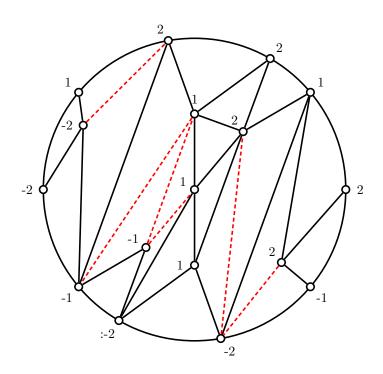
# International Conference on Graph Theory, Combinatorics and Applications

June 17–21, 2019

National Sun Yat-sen University Kaohsiung, Taiwan





動棒壘球場 Baseball Field

■ 溜冰場 Skating Rink

©2015 National Sun Yat-sen University, All Rights Reserved.

- 幼稚園 Kindergarten
- 西子灣沙灘會館 Sunset Beach Resort
- ◎ 周大觀藝術銅雕 Artistic Bronze of Chou Ta-Kuan
- 圆 西子樓 Sizih Building (Alumni Service Center)









#### 1 Schedule

- \* Venue for every talk: Theater, College of Science 1F
- \* 60 minutes = 50 minutes (presentation) + 10 minutes (questions)
- \* 30 minutes = 25 minutes (presentation) + 5 minutes (questions)

### June 17, Monday

**17:00–19:00** Registration

### June 18, Tuesday

**09:00–09:10** Opening remarks

**Session 1.** 09:10–10:40

09:10–10:10 Jarik Nešetřil, DENSITY, DUALITY AND UNIVERSALITY

10:10-10:40 Eckhard Steffen, Component factors of simple edge chromatic critical graphs

 $Tea\ break$ 

**Session 2.** 11:00–12:30

11:00-11:30 Kenta Ozeki, Spanning trees with few leaves in graphs on surfaces

11:30-12:00 Arnaud Pecher, On the density of sets of the Euclidean plane avoiding distance 1

12:00-12:30 Zhibin Du, Asymptotic normality criteria of coefficients of a polynomial and their applications in combinatorics

Lunch

**Session 3.** 13:20–15:20

13:20–14:20 Zdeněk Dvořák, Splitter and Baker games

14:20-14:50 Daphne Der-Fen Liu, Improved Lower Bounds for Radio Number of Trees

14:50–15:20 Jakub Kozik, Shattering and extreme hypergraphs for Property B

Tea break

**Session 4.** 15:40–17:10

15:40–16:10 Andrzej Dudek, Powers of Hamiltonian cycles in randomly augmented graphs

16:10-16:40 Pawel Pralat, Thresholds in random graphs with focus on thresholds for k-regular subgraphs

16:40–17:10 Hao Qi, The fault-diameter and wide-diameter of twisted hypercubes

**18:00** Reception (College of Science 4F)

### June 19, Wednesday

**Session 1.** 09:00–10:30

09:00-10:00 Martin Škoviera, Small snarks with oddness 4: construction and classification

**10:00–10:30** Ringi Kim, 1-join and polynomial  $\chi$ -boundedness

 $Tea\ break$ 

**Session 2.** 10:50–12:20

10:50–11:20 Boram Park, On path induced saturation problem

11:20-11:50 Shinya Fujita, A short survey on safe set problems in graphs

11:50–12:20 Hong-Bin Chen, A Cute Result Beyond Bertrand's Postulate

Group photo

Lunch

**Session 3.** 13:20–15:20

13:20-14:20 Patrice Ossona de Mendez, First-Order Interpretations of Bounded Expansion Classes

14:20–14:50 Jing Huang, Graph and digraph classes arising from list homomorphism problems

 $\textbf{14:50-15:20} \hspace{0.2cm} \textbf{Tomasz} \hspace{0.1cm} \textbf{Krawczyk}, \hspace{0.1cm} \textit{Testing isomorphism of circular-arc graphs} - \textit{Hsu's approach revisited} \\$ 

 $Tea\ break$ 

**Session 4.** 15:40–17:10

15:40–16:10 Peng-An Chen, The chromatic number of general Kneser hypergraphs

16:10–16:40 Grzegorz Gutowski, Online Coloring of Short Intervals

**16:40–17:10** Daqing Yang, A few applications of generalized coloring numbers

18:00 Banquet (Sunset Beach Resort)

## June 20, Thursday

**Session 1.** 09:00-10:30

 $\mathbf{09:00-10:00}$  Gyula O.H. Katona, The domination number of the graph defined by two levels of the n-cube

10:00-10:30 David Kuo, Transforable domination number of graphs

 $Tea\ break$ 

**Session 2.** 10:50–12:20

10:50-11:20 Hsin-Hao Lai, Results on Sigma Colorings of Halin Graphs

11:20-11:50 Seog-Jin Kim, The Alon-Tarsi number of subgraphs of a planar graph

11:50–12:20 Uwe Schauz, The Combinatorial Nullstellensatz, Jaeger's Conjecture and the Four Color Problem

Lunch

Excursion (Location: TBA)

## June 21, Friday

**Session 1.** 09:00–10:30

09:00-09:30 Chih-wen Weng, On degrees and average 2-degrees in graphs

09:30-10:00 Hau-Wen Huang, Ramanujan graphs and sequences

10:00-10:30 Hong-Gwa Yeh, On matrix equations rising in network analysis

 $Tea\ break$ 

**Session 2.** 10:50–12:20

10:50–11:20 Yeong-Nan Yeh, Combinatorial enumeration and context-free grammars

11:20–11:50 Sen-Peng Eu, Some results on permutation statistics

11:50–12:20 Wei-Hsuan Yu, Recent progress on equiangular line problems

Lunch

**13:50–15:50** Discussion

### 2 Plenary Lectures (50 minutes)

### Splitter and Baker games

Zdeněk Dvořák, Charles University

 $\uparrow$ 

A number of interesting graph properties can be described in terms of games, a well-known example being the cop-robber game characterization of treewidth. Splitter game was recently introduced to characterize nowhere-dense graph classes and used as a tool to prove tractability of the first-order model checking in these classes. Baker game follows a similar idea to generalize Baker's approximation technique from planar graphs to much more general graph classes. We will explore and survey results on these games.

# The domination number of the graph defined by two levels of the n-cube

Gyula O.H. Katona, MTA Rényi Institute

 $\uparrow$ 

Consider all k-element subsets and  $\ell$ -element subsets  $(k > \ell)$  of an n-element set as vertices of a bipartite graph. Two vertices are adjacent if the corresponding  $\ell$ -element set is a subset of the corresponding k-element set. Let  $G_{k,\ell}$  denote this graph. The domination number of  $G_{k,1}$  is exactly determined. We also prove that  $\gamma(G_{k,2})$  is asymptotically equal to

$$\frac{k+3}{2(k-1)(k+1)}n^2$$

for  $k \geq 3$ . The upper estimate is proved by a random construction. We also suggest a way to find a deterministic construction, but it is completed only for k = 3 and 4.

Joint work with Leila Badakhshian and Zsolt Tuza.

### DENSITY, DUALITY AND UNIVERSALITY

Jarik Nešetřil, Charles University

 $\uparrow$ 

The study of colourings is closely related to properties of homomorphism order. This also displays striking difference between undirected and oriented graohs. Extending (by now classical) results of X. Zhu and author we recently proved density for oriented trees. We survey related universality and duality results.

### First-Order Interpretations of Bounded Expansion Classes

Patrice Ossona de Mendez, French National Centre for Scientific Research

 $\uparrow$ 

The notion of bounded expansion captures uniform sparsity of graph classes and renders various algorithmic problems that are hard in general tractable. In particular, the model-checking problem for first-order logic is fixed-parameter tractable over such graph classes. With the aim of generalizing such results to dense graphs, we introduce classes of graphs with structurally bounded expansion, defined as first-order interpretations of classes of bounded expansion. As a first step towards their algorithmic treatment, we provide their characterization analogous to the characterization of classes of bounded expansion via low treedepth decompositions, replacing treedepth by its dense analogue called shrubdepth.

#### Small snarks with oddness 4: construction and classification

Martin Škoviera, Comenius University

 $\uparrow$ 

The oddness of a cubic graph is the smallest number of odd circuits in a 2-factor of the graph. This invariant is widely considered to be one of the most important measures of uncolourability of cubic graphs

and as such has been repeatedly reoccurring in numerous investigations of problems and conjectures surrounding snarks (connected cubic graphs admitting no proper 3-edge-colouring). For example, snarks with oddness at least 4 or higher can serve as potential counterexamples to several important conjectures in graph theory (such as Fulkerson's conjecture, Fan-Raspaud conjecture, 5-flow conjecture, and others). On the other hand snarks with oddness at least 4 are very difficult to find.

In the talk we shall deal with the problem of determining the smallest order of a nontrivial snark (one which is cyclically 4-edge-connected and has girth at least 5) of oddness at least 4. Using a combination of structural analysis with extensive computations we prove that the smallest order of a snark with oddness at least 4 and cyclic connectivity 4 is 44. We further show that there are exactly 31 such snarks, all of them built up from subgraphs of the Petersen graph and a small number of additional vertices. We explain the reasons why these snarks have oddness 4 and sketch the proof that they form the complete set of snarks with cyclic connectivity 4 and oddness 4 on 44 vertices. We conclude with a number of related open problems.

#### 3 25-minute Talks

### A Cute Result Beyond Bertrand's Postulate

Hong-Bin Chen, National Chung Hsing University

 $\uparrow$ 

Bertrand's postulate assures that for any positive integer n > 3 there exists a prime p between n and 2n. A consequence of Bertrand's postulate states that the set of integers  $\{1,2,\ldots,2n\}$  can be partitioned into pairs so that the sum of each pair is a prime number for any positive integer n. In this talk, I will introduce its proof and a stronger conjecture by Filz in 1982 that the set of integers  $\{1,2,\ldots,2n\}$  can be rearranged into a cycle so that the sum of any two adjacent integers is a prime number. With a fundamental result in graph theory and a recent breakthrough on the twin prime conjecture, we prove that Filz's conjecture is true for infinitely many cases. This talk is based on a joint work with Hung-Lin Fu and Jun-Yi Guo.

### The chromatic number of general Kneser hypergraphs

Peng-An Chen, National Taitung University

 $\uparrow$ 

Let  $\mathcal{H}$  be a family of sets and define the r-uniform general Kneser hypergraph  $\mathrm{KG}^r(\mathcal{H})$  on vertex set  $\mathcal{H}$  with a hyperedge for any collection of r pairwise disjoint elements of  $\mathcal{H}$ . The chromatic numbers of these Kneser hypergraphs have been of fundamental interest since Lovász's 1978 proof of Kneser's conjecture (concerning r=2, and F the family of k-subsets of [n]).

In this talk, we investigate the chromatic number of general Kneser hypergraphs considered by Sani and Alishahi [2], and Aslam, Chen, Coldren, Frick, and Setiabrata [1].

- [1] Jai Aslam, Shuli Chen, Ethan Coldren, Florian Frick, and Linus Setiabrata, On the generalized Erdős-Kneser conjecture: Proofs and reductions. J. Comb. Theory, Ser. B **135** (2019), 227-237.
- [2] Roya Abyazi Sani and Meysam Alishahi, A new lower bound for the chromatic number of general Kneser hypergraphs, Eur. J. Comb. 71 (2018), 229-245.

### Asymptotic normality criteria of coefficients of a polynomial and their applications in combinatorics

Zhibin Du, Academia Sinica

 $\uparrow$ 

The asymptotic distribution theory for coefficients of a polynomial is an active topic in asymptotic analysis. In 1967, Harper proposed a criterion to measure the asymptotic normality of a series of numbers, when he researched the asymptotic behavior of Stirling numbers of the second kind.

In this talk, we will discuss some further asymptotic normality criteria of coefficients of a polynomial with all real roots or purely imaginary roots (including 0). These new asymptotic normality criteria turn out to be very efficient and have abundant applications in combinatorics, mainly including the coefficients of a series of characteristic polynomials of adjacency matrix, Laplacian matrix, signless Laplacian matrix, skew-adjacency matrix, chromatic polynomial, and some graph numbers, such as matching numbers, independence numbers, clique numbers. Among which, we generalize and verify some conjectures about asymptotic normality in combinatorics, e.g., the matching numbers proposed by Godsil and Kahn, the (signless) Laplacian coefficients claimed by Wang et al..

### Powers of Hamiltonian cycles in randomly augmented graphs

Andrzej Dudek, Western Michigan University

 $\uparrow$ 

It follows from the theorems of Dirac and of Komlós, Sarközy, and Szemerédi, who confirmed the Posá-Seymour conjecture, that for every  $k \geq 1$  and sufficiently large n already the minimum degree  $\delta(G) \geq \frac{k}{k+1}n$  for an n-vertex graph G alone suffices to ensure the existence of the k-th power of a

Hamiltonian cycle. In this talk we will determine the number of random edges one has to add to a graph G with minimum degree  $\delta(G) \ge \left(\frac{k}{k+1} + \varepsilon\right) n$  (with  $\varepsilon > 0$ ) in order to create an  $\ell$ -th power of a Hamiltonian cycle, where  $\ell \ge k+1$ .

This is joint work with Sylwia Antoniuk, Christian Reiher, Andrzej Ruciński and Mathias Schacht.

### Some results on permutation statistics

Sen-Peng Eu, National Taiwan Normal University

 $\uparrow$ 

The study of permutation statistics is a central topic in enumerative and algebraic combinatorics and is still very active. In this talk we present some new results and conjectures on permutation statistics.

### A short survey on safe set problems in graphs

Shinya Fujita, Yokohama City University

 $\uparrow$ 

In [S. Fujita, G. MacGillivray, T. Sakuma: Safe set problem on graphs. Discrete Applied Math. 215: 106-111 (2016)], the authors defined a safe set in a graph G = (V(G), E(G)) as a set S of vertices of G with the property that  $|V(C)| \ge |V(D)|$  for every component C of the subgraph G[S] of G induced by S and every component S of the subgraph S of S such that some vertex in S is adjacent to some vertex in S. For convenience, we call two disjoint subgraphs S and S of S adjacent if some vertex in S is adjacent to some vertex in S.

Naturally, we can consider the weighted version of safe sets in vertex-weighted graphs. For a graph G and a weight function  $w:V(G)\to Z\geq 0$ , we consider the vertex weighted graph (G,w). For a set U of vertices of G, let  $w(U)=\sum_{u\in U}w(u)$ . A set S of vertices of G is a weighted safe set in G if  $w(C)\geq w(D)$  for every component C of G[S] and every component D of G such that D is adjacent to C.

These notion on safe sets attracts much attention and a lot of work has been done so far. Some recent results in this topic will be reviewed.

### Online Coloring of Short Intervals

Grzegorz Gutowski, Jagiellonian University

⇑

We study the online graph coloring problem restricted to the intersection graphs of intervals with lengths in  $[1,\sigma]$ . For  $\sigma=1$  it is the class of unit interval graphs, and for  $\sigma=\infty$  the class of interval graphs. Our focus is on intermediary classes. We present a  $(1+\sigma)$ -competitive algorithm, which beats the state of the art for  $1<\sigma<2$ . On the lower bound side, we prove that no algorithm is better than 5/3-competitive for any  $\sigma>1$ , nor better than 7/4-competitive for any  $\sigma>2$ , and that no algorithm beats the 5/2 asymptotic competitive ratio for all, arbitrarily large, values of  $\sigma$ .

#### Ramanujan graphs and sequences

Hau-Wen Huang, National Central University

1

A finite connected undirected (q+1)-regular graph is called Ramanujan if the absolute value of each nontrivial eigenvalue is less than or equal to  $2\sqrt{q}$ . In this talk we display a sequence  $\{h_k\}_{k=1}^{\infty}$  obtained from the Ihara zeta function. Moreover, we characterize the Ramanujan graph via the positivity of  $\{h_k\}_{k=1}^{\infty}$ .

#### Graph and digraph classes arising from list homomorphism problems

Jing Huang, University of Victoria

 $\uparrow$ 

Fix a graph H, the list homomorphism problem for H asks whether a graph G together with lists  $L(v) \subseteq V(H), v \in V(G)$  admits a homomorphism  $f:G \to H$  such that  $f(v) \in L(v), v \in V(G)$ . List homomorphism problems for digraphs are defined analogously. The dichotomy of list homomorphism problems for graphs is well-understood. In particular, we understand exactly the graphs for which list homomorphism problems are polynomial time solvable. These include several well-studied classes of graphs such as interval graphs, the complements of 2-clique circular arc graphs, and bi-arc graphs. However, the digraph classes which define the dichotomy of list homomorphism problems for digraphs are not completely comprehended. We exhibit several classes of digraphs which share some common forbidden structures and for which the list homomorphism problems are polynomial time solvable. Surprisingly, one of the forbidden structures gives rise to beautiful bipartite analogues of the classical comparability and cocomparability graphs. This contains joint work with P. Hell, T. Feder, J. Lin, R. McConnell, and A. Rafiev.

### 1-join and polynomial $\chi$ -boundedness

Ringi Kim, Korea Advanced Institute of Science and Technology (KAIST)

 $\uparrow$ 

A class  $\mathcal{G}$  of graphs is  $\chi$ -bounded if there is a function f such that for every graph  $G \in \mathcal{G}$  and every induced subgraph H of G,  $\chi(G) \leq f(\omega(H))$ . In addition, we say that G is polynomially  $\chi$ -bounded if f can be taken as a polynomial function.

It was proved by Dvořák and Král, and Kim independently that if a class  $\mathcal{G}$  of graphs is  $\chi$ -bounded, then the closure of  $\mathcal{G}$  under 1-joins is also  $\chi$ -bounded. In this talk, we show that if a class  $\mathcal{G}$  of graphs is polynomially  $\chi$ -bounded, then so is the closure of  $\mathcal{G}$  under 1-joins.

### The Alon-Tarsi number of subgraphs of a planar graph

Seog-Jin Kim, Konkuk University

 $\uparrow$ 

This paper constructs a planar graph  $G_1$  such that for any subgraph H of  $G_1$  with maximum degree  $\Delta(H) \leq 3$ ,  $G_1 - E(H)$  is not 3-choosable, and a planar graph  $G_2$  such that for any star forest F in  $G_2$ ,  $G_2 - E(F)$  contains a copy of  $K_4$  and hence  $G_2 - E(F)$  is not 3-colourable. On the other hand, we prove that every planar graph G contains a forest F such that the Alon-Tarsi number of G - E(F) is at most 3, and hence G - E(F) is 3-paintable and 3-choosable. This is joint with Ringi Kim and Xuding Zhu.

### Shattering and extreme hypergraphs for Property B

Jakub Kozik, Jagiellonian University

1

How many edges do we need in order to construct a k-graph (i.e. a k-uniform hypergraph) which is not two colorable? This numbers, denoted by m(k), have been intensively studied since their introduction by Erdős and Hajnal in 1961. As a result, the known lower bounds have been improved a number of times and nowadays we know that  $m(k) = \Omega(\sqrt{k/\log(k)} \ 2^k)$  (Radhakrishnan and Srinivasan 2000). Interestingly the story of the upper bounds is much shorter – bound  $m(k) = O(k^2 \ 2^k)$  obtained by Erdős in 1964 has not been improved since. That bound is a consequence of the observation that, for some specific constant c, random k-graph built on a set of vertices of size  $n = \Theta(k^2)$  with high probability is not two colorable when the number of edges is of the order  $ck^2 2^k$ . Trying to gain some insight into the structure of the hypergraphs that witness the upper bound for m(k), we analyze random k-graphs with polynomial (in k) number of vertices. We focus mainly on the evolution of the space of proper colorings and algorithms that performs well in a random case. Our developments are largely inspired by analogous considerations within the framework of constraint satisfaction problems for random instances, where k is considered constant and the number of vertices n tends to infinity.

# Testing isomorphism of circular-arc graphs – Hsu's approach revisited

Tomasz Krawczyk, Jagiellonian University

 $\uparrow$ 

Circular-arc graphs are intersection graphs of arcs on the circle. The aim of our work is to present a polynomial time algorithm testing whether two circular-arc graphs are isomorphic. To accomplish our task we construct decomposition trees, which are the structures representing all normalized intersection models of circular-arc graphs. Normalized models reflect the neighbourhood relation in circular-arc graphs and can be seen as their canonical representations; in particular, every intersection model can be easily transformed into a normalized one.

Our work adapts and appropriately extends the previous work on the similar topic done by Hsu [SIAM J. Comput. 24(3), 411-439, (1995)]. In his work, Hsu developed decomposition trees representing all normalized models of circular-arc graphs. However due to the counterexample given in [Discrete Math. Theor. Comput. Sci., 15(1), 157-182, 2013], his decomposition trees can not be used by algorithms testing isomorphism of circular-arc graphs.

### Transforable domination number of graphs

David Kuo, National Don Hwa University

 $\uparrow$ 

Let G be a connected graph, and let  $\mathcal{D}(G)$  be the set of all dominating (multi)sets for G. For  $D_1$  and  $D_2$  in  $\mathcal{D}(G)$ , we say that  $D_1$  is single-step transferable to  $D_2$ , denoted as  $D_1 \longrightarrow D_2$ , if there exist  $u \in D_1$  and  $v \in D_2$ , such that  $uv \in E(G)$  and  $D_1 - \{u\} = D_2 - \{v\}$ . We write  $D_1 \stackrel{*}{\longrightarrow} D_2$  if  $D_1$  can be transferred to  $D_2$  through a sequence of single-step transfers. We say that G is k-transferable if  $D_1 \stackrel{*}{\longrightarrow} D_2$  for any  $D_1$ ,  $D_2 \in \mathcal{D}(G)$  with  $|D_1| = |D_2| \ge k$ . The transforable domination number of G, denoted by  $\gamma_{t^*}(G)$ , is the smallest integer k such that G is k-transforable. In this talk, some results concerning the transforable domination number of graphs, will be given.

### Results on Sigma Colorings of Halin Graphs

Hsin-Hao Lai, National Kaohsiung Normal University

 $\uparrow$ 

Let G be a simple graph. A  $sigma\ coloring$  of G is an assignment of integers to the vertices of G such that for every two adjacent vertices the sums of integers assigned to their neighbors are different. The  $sigma\ chromatic\ number$  of G is the minimum number of integers required in a sigma coloring of G.

A Halin graph is a plane graph  $G = T \cup C$  constructed as follows. Let T be a tree of order at least 4 and no vertex of T is of degree 2. And let C be a cycle connecting the leaves of T in such a way that C forms the boundary of the unbounded face.

In this talk, I will present new results on sigma colorings of Halin graphs.

### Improved Lower Bounds for Radio Number of Trees

Daphne Der-Fen Liu, California State University Los Angeles

1

Let G be a graph with diameter d, and let k be a positive integer. A radio labeling of G is a function,  $f:V(G)\to\{0,1,2,\ldots\}$ , such that for any two vertices  $u,v\in V(G)$ , it holds that  $|f(u)-f(v)|\geq d+1-d(u,v)$ , where d(u,v) is the distance between u and v. The span of f is the difference of the largest and smallest values of f(V). The radio number of G, denoted as rn(G), is the minimum span among all radio labelings admitted by G. For a tree T with n vertices and diameter d, it is known [4] that  $rn(T)\geq (n-1)(d+1)-2w(T)+1$ , where w(T) is the weight of T. We call a tree with radio number equal to this lower bound a lower-bound tree. Many families of trees have been proved to be lower bound trees (cf. [1,2,3,4,5]). We establish general properties of lower bound trees, and device a method for creating new lower bound trees by combining two lower bound trees, or by attaching a non-trivial tree to a lower bound tree. In addition. Our results can be applied to obtain shorter proofs of several known families of lower bound trees. For non-lower-bound trees, we give improved lower bounds and show that

the new bound is sharp for some known families of trees (including odd paths [5] and complete binary trees [3]) and many new families of trees.

This talk will include joint works with A. Chavez, S. Das, L. Saha, and M. Shurman.

- [1] D. Bantva, S. Vaidya, S. Zhou, Radio number of trees, Discrete Applied Mathematics 317 (2017) 110–122.
- [2] V. Halásza and Z. Tuza, Distance-constrained labeling of complete trees, Discrete Math. 338 (2015) 1398–1406.
- [3] X. Li, V. Mak, S. Zhou, Optimal radio labellings of complete m-ary trees, Discrete Appl. Math. 158 (2010) 507–515.
- [4] D. D. -F. Liu, Radio number for trees, Discrete Math. 308 (2008) 1153–1164.
- [5] D. D. -F. Liu, X. Zhu, Multi-level distance labelings for paths and cycles, SIAM Jornal on Discrete Math. 19 (2005) 610–621.

#### Spanning trees with few leaves in graphs on surfaces

Kenta Ozeki, Yokohama National University

 $\uparrow$ 

In a graph G, a cycle or a path is **Hamiltonian** if it contains all vertices of G. In 1956, Tutte proved that every 4-connected planar graph is Hamiltonian. Since planar graphs can be regarded as graphs on the sphere, it is natural to think about graphs on higher genus surfaces. With this direction, the most attractive conjecture is due to Nash-Willams and Grünbaum, which says that every 4-connected graph on the torus is Hamiltonian. This conjecture is still open.

Note that for any such a surface  $F^2$  with genus more than the torus, there exist infinitely many 4-connected non-Hamiltonian graphs on  $F^2$ . However, we can expect the existence of some structures which have weaker (but still interesting) property than the Hamiltonicity. For example, it is unknown that whether every 4-connected graph on the nonorientable surface of crosscap number 3 has a Hamiltonian path. Similarly, the author conjectured that for any surface of Euler characteristic  $\chi$ , there exists a spanning tree with at most  $O(-\chi)$  leaves. In this talk, we will give a recent result concerning the conjecture.

This work is a joint work with Atsuhiro Nakamoto (Yokohama National University).

### On path induced saturation problem

Boram Park, Ajou University

 $\uparrow$ 

For a graph H, a graph G is called H-induced-saturated if G does not contain a copy of H as an induced subgraph, but removing any edge from G or adding any non-edge of G to G always creates an induced copy of H. Martin and Smith (2012) showed that there does not exist a  $P_4$ -induced-saturated graph, where  $P_n$  is the path on n vertices. Recently, Axenovich and Csikós (2018) studied related questions, and asked if there exists a  $P_n$ -induced-saturated graph for any  $n \geq 5$ , and it was shown that there exists a  $P_6$ -induced-saturated graph by Räty (2018). In this paper, we present some results on  $P_n$ -induced-saturated graphs. More precisely, we find a  $P_{3n}$ -induced-saturated graph for every positive integer n, and also find infinitely many  $P_6$ -induced-saturated graph for every  $n \geq 5$ . We also asks some open questions. This is based on joint work with Ilkyoo Choi and Eun Kyung Cho.

## On the density of sets of the Euclidean plane avoiding distance

Arnaud Pecher, Bordeaux University

1

A subset  $A \subset \mathbb{R}^2$  is said to avoid distance 1 if:  $\forall x, y \in A, ||x - y||_2 \neq 1$ . We study the number  $m_1(\mathbb{R}^2)$  which is the supremum of the upper densities of measurable sets avoiding distance 1 in the

Euclidean plane. Intuitively,  $m_1(\mathbb{R}^2)$  represents the highest proportion of the plane that can be filled by a set avoiding distance 1. This parameter is related to the fractional chromatic number  $\chi_f(\mathbb{R}^2)$  of the plane. We establish that  $m_1(\mathbb{R}^2) \leq 0.25646$  and  $\chi_f(\mathbb{R}^2) \geq 3.8992$ . Joint work with Thomas Bellitto and Antoine Sedillot.

# Thresholds in random graphs with focus on thresholds for k-regular subgraphs

Pawel Pralat, Ryerson University

 $\uparrow$ 

The most intriguing discovery made by Erdős and Rényi in random graphs is the phenomenon of thresholds. For many graph properties the limiting probability that a random graph possesses them jumps from 0 to 1 (or vice versa) very rapidly, that is, with a rather small increase in the (expected) number of edges. We present a few classical and well understood examples but then move to new results. We prove that the binomial random graph  $G_{n,p=c/n}$  with high probability has a k-regular subgraph if c is at least  $e^{-\Theta(k)}$  above the threshold for the appearance of a subgraph with minimum degree at least k; i.e. an non-empty k-core. In particular, this pins down the threshold for the appearance of a k-regular subgraph to a window of size  $e^{-\Theta(k)}$ .

### The fault-diameter and wide-diameter of twisted hypercubes

Hao Qi, Zhejiang Normal University and Academia Sinica

 $\uparrow$ 

A twisted hypercube of dimension n is created from two twisted hypercubes of dimension n-1 by adding a matching joining their vertex sets, with the twisted hypercube of dimension 0 consisting of one vertex and no edges. In this talk, we introduce three special twisted hypercubes:  $Z_{n,k}$ ,  $H_n$  and  $RQ_n$  (random twisted hypercube) by generating special matchings at each step. And show that (asymptotically almost surely) they have asymptotically optimal fault-diameter and wide-diameter. This is joint work with Andrzej Dudek, Xavier Pérez-Giménez, Pawel Prałat, Douglas West and Xuding Zhu.

# The Combinatorial Nullstellensatz, Jaeger's Conjecture and the Four Color Problem

Uwe Schauz, Xian Jiaotong-Liverpool University

 $\uparrow$ 

We start by introducing and explaining the Quantitative Combinatorial Nullstellensatz, a strengthened version of Alon's Combinatorial Nullstellensatz. A corollary of this theorem is often used to tackle list coloring problems. In this talk, however, we apply it to Jaeger's conjecture, which says that every non-singular  $n \times n$  matrix A, over a finite field  $\mathbb{F}_q$  with q > 3 elements, has a nowhere zero point. Here, a nowhere zero point of A is a vector x in  $\mathbb{F}_q^n$  such that neither x nor Ax has a zero entry. Alon and Tarsi could prove this conjecture for all non-prime cardinalities q. We give a new short proof of their partial result. Afterwards, we discuss a connection between nowhere zero points and the four color problem.

## Component factors of simple edge chromatic critical graphs

Eckhard Steffen, Paderborn University

 $\uparrow$ 

There are many (open) conjectures on component factors of simple edge-chromatic critical graphs. The talk surveys the topic and present some new results. In particular it is shown that every edge-chromatic critical has a [1, 2]-factor.

### On degrees and average 2-degrees in graphs

Chih-wen Weng, National Chiao Tung University

Let G be a simple graph without isolated vertices. For a vertex i in G, the degree  $d_i$  is the number of vertices adjacent to i and the average 2-degree  $m_i$  is the mean of the degrees of the vertices which are adjacent to i. The sequence of pairs  $(d_i, m_i)$  is called the sequence of degree pairs of G. We provide some necessary conditions for a sequence of real pairs  $(a_i, b_i)$  of length n to be the degree pairs of a graph of order n. A graph G is called pseudo k-regular if  $m_i = k$  for every vertex i while  $d_i$  is not a constant. Let N(k) denote the minimum number of vertices in a pseudo k-regular graph. We utilize the above necessary conditions to find all pseudo 3-regular graphs of orders no more than 10, and all pseudo k-regular graphs of order N(k) for k up to 7. In general for  $k \geq 2$ , we show that if k is odd then  $k+3 \leq N(k) \leq k+4$  and both bounds are tight; and if k is even then  $k+3 \leq N(k) \leq k+6$ . This is a joint work with Yu-pei Huang and Chia-an Liu.

### A few applications of generalized coloring numbers

Daqing Yang, Zhejiang Normal University

 $\uparrow$ 

 $\uparrow$ 

The generalized coloring numbers  $wcol_r(G)$  and  $scol_r(G)$  of a graph G were introduced by Kierstead and Yang as a generalization of the usual coloring number col(G), and have since found interesting theoretical and algorithmic applications. Zhu characterized graph classes with bounded expansion as those classes  $\mathcal{C}$  for which there is a function  $f: \mathbb{Z}^+ \to \mathbb{Z}^+$  such that all graphs  $G \in \mathcal{C}$  and all integers  $r \in \mathbb{N}$  satisfy  $scol_r(G) \leq f(r)$ . We shall talk about a few applications of the generalized coloring numbers.

Let  $G^p$  denote the p-th power of G. We obtain some upper bounds for  $col(G^p)$ . We show that the generalized coloring numbers can be used to study a form of generalized oriented chromatic numbers  $\chi_o^k$ ; and showing that classes with bounded expansion can be defined by means of boundedness of generalized oriented chromatic numbers  $\chi_o^k$ .

The talk is based on some joint work with Prof. Kierstead, Prof. Nešetřil, and Prof. Zhu.

### On matrix equations rising in network analysis

Hong-Gwa Yeh, National Central University

 $\uparrow$ 

In this talk, several matrix equations together with their applications to electrical network analysis and social network analysis are presented.

### Combinatorial enumeration and context-free grammars

Yeong-Nan Yeh, Academia Sinica

 $\uparrow$ 

In this talk, we will show many combinatorial polynomials can be generated by context-free grammars. We present combinatorial models and a unified refinement of these combinatorial polynomials . We also introduce two types of Jacobi-pairs of differential equations. We present a general method to derive the solutions of these differential equations. We then study the combinatorial, algebraic properties for these polynomials.

### Recent progress on equiangular line problems

Wei-Hsuan Yu, National Central University

⇑

I will talk about the history of equiangular line problems and present the recent progress on this topics.