

# Extensions of Gallai's theorem on colour-critical graphs

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## Abstract

In 1963 Gallai published two seminal papers on colour-critical graphs: *Kritische Graphen I* and *II*. In the second of these papers Gallai proved that a  $k$ -critical graph with a connected complement has at least  $2k - 1$  vertices. So any  $k$ -critical graph with less than  $2k - 1$  vertices is the complete join of two smaller critical graphs. Gallai's original proof was quite long and involved the Edmonds-Gallai decomposition.

I will show a different proof of Gallai's theorem which is much shorter. I will then show how the theorem relates to factor-critical graphs,  $t$ -stable hypergraphs and tau-critical hypergraphs. Finally, I will discuss graphs which achieve equality in Gallai's theorem.