The nullity and maximum nullity of a graph

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Abstract

Given a graph G (simple, undirected, finite) on n vertices and a field F, the maximum nullity of G over F, denoted by $M^F(G)$, is the largest possible nullity over all $n \times n$ symmetric matrices over F whose (i, j)th entry (for $i \neq j$) is nonzero whenever ij is an edge in G and is zero otherwise. The minimum rank of G over F is $\mathrm{mr}^F(G) = n - M^F(G)$. The maximum nullity/minimum rank problem of a graph G is to determine $M^F(G)$ (or equivalently, $\mathrm{mr}^F(G)$). This problem and its variations have received considerable attention over the years. In [AIM Minimum Rank–Special Graphs Work Group, Zero forcing sets and the minimum rank of graphs, Linear Algebra Appl. 428 (2008) 1628-1648], a new graph parameter $Z(G)$, the zero forcing number, was introduced to bound $M^F(G)$ from above. The authors posted an attractive question: What is the class of graphs G for which $Z(G) = M^F(G)$ for some field F? In the first part of this talk, I'll present our research results on the above question. We make significant advances in the maximum nullity/minimum rank problem by determining maximum nullity for several large families of graphs and showing maximum nullity is equal to zero forcing number.

The nullity of a graph G is the number of zero eigenvalues of the adjacency matrix of G. In the second part of this talk, we consider the following problem: What is the structure of an n-vertex connected graph with nullity $n - 4$? This question has not yet been fully answered in the literature, and only some partial results are known. We resolve this question by completely characterizing the structure of n-vertex connected graphs with nullity $n - 4$. If time permits, I will also talk about our recent work done on the rank of a cograph.

This talk is based on joint works with Liang-Hao Huang (National Central University, Taiwan) and Gerard J. Chang (National Taiwan University, Taiwan).