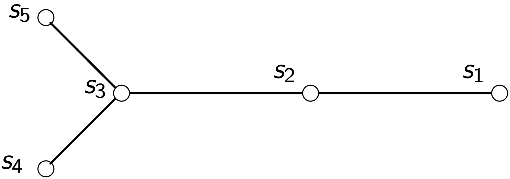
Lit-only σ-game on nondegenerate graphs

Hau-wen Huang

Mathematics Division, National Center for Theoretical Sciences

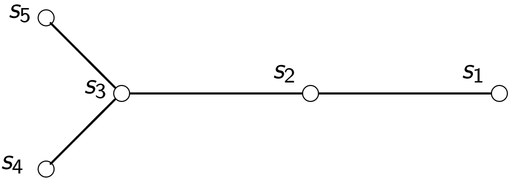
Lit-only σ-game

Example: Let Γ be



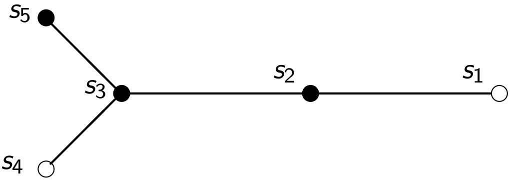
Lit-only σ-game

Example: The lit-only σ-game on Γ



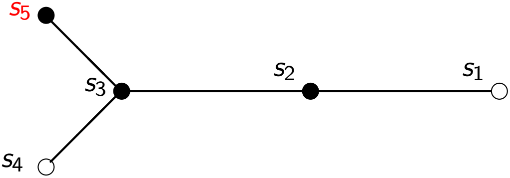
Lit-only σ-game

Example: The lit-only σ-game on Γ



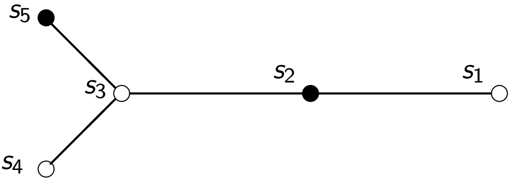
Lit-only σ-game

Example: The lit-only σ-game on Γ



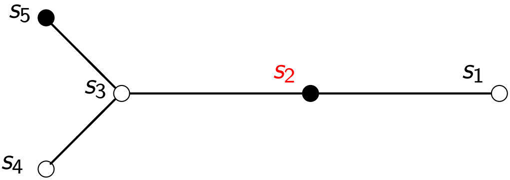
Lit-only σ-game

Example: The lit-only σ-game on Γ



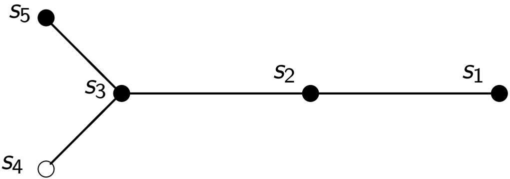
Lit-only σ-game

Example: The lit-only σ-game on Γ



Lit-only σ-game

Example: The lit-only σ-game on Γ



Lit-only σ-game

A configuration of the lit-only σ-game on a finite graph Γ is an assignment of one of two states, *on* or *off*, to all vertices of Γ.
Given a configuration, a move of the lit-only σ-game on Γ allows the player to choose an *on* vertex s of Γ and change the states of all neighbors of s.
Given an initial configuration, the goal is to minimize the number of *on* vertices of Γ by a finite sequence of moves of the Reeder's game on Γ.

The story began that ...

Prof. Chang introduced the work of Prof. Chuah on Vogan diagrams to combinatorists.

k-lit graphs

Given an integer k, we say that the underlying graph is *k-lit* if, for any configuration, the number of *on* vertices can be reduced to at most k by a finite sequence of moves of the lit-only σ-game on Γ.

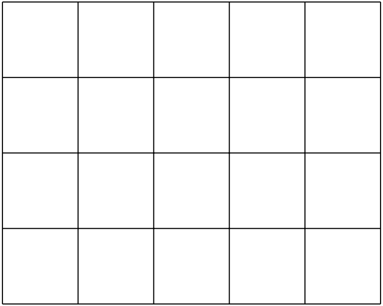
The minimum light number of a graph

Let Γ be a graph. The *minimum light number* of Γ is the smallest integer k for which Γ is k-lit.

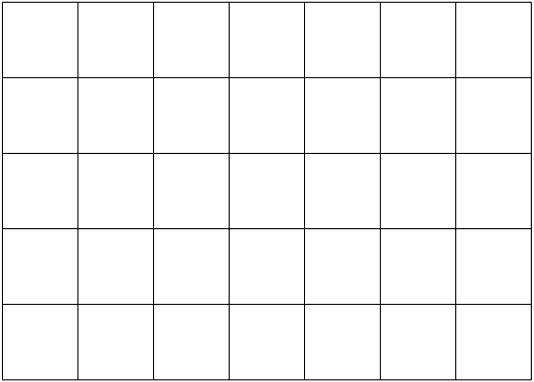
1-lit graphs

A known result:
All trees with perfect matchings are 1-lit.

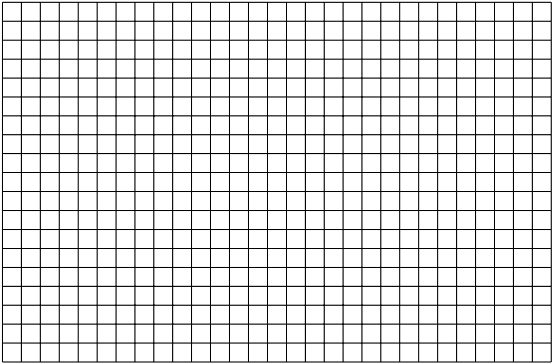
1-lit graphs



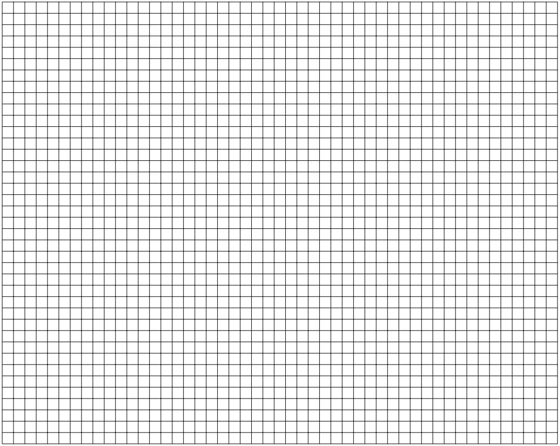
1-lit graphs



1-lit graphs



1-lit graphs



1-lit graphs

All m by n girds with $g.c.d.(m+1, n+1) = 1$ are 1-lit.

Graphs with odd number of perfect matchings

My new result:

Theorem

Assume that Γ is a simple graph which is not a line graph and with an odd number of perfect matchings. Then Γ is 2-lit. Moreover we give a linear algebraic criterion for Γ to be 1-lit.

Bipartite graphs with odd number of perfect matchings

A consequence of my new result:

Corollary

A bipartite graph Γ with an odd number of perfect matchings are 2-lit. Moreover Γ is 1-lit if and only if Γ is an edge or contains a vertex with even degree.

Graphs with odd number of perfect matchings

Let Γ be a simple graph. Then the following are equivalent:

1. The number of perfect matchings in Γ is odd.
2. The determinant of the adjacent matrix of Γ is odd.

Suppose that 1, 2 hold. Based on some reason, we call such a graph Γ is a *nondegenerate graph*.

Line graphs with odd number of perfect matchings

Let Γ be a line graph. Then the following are equivalent:

1. The number of perfect matchings in Γ is odd.
2. Γ is a claw-free block graph of even order.
3. Γ is a line graph of an odd-order tree.

Line graphs with odd number of perfect matchings

Let Γ be a line graph. Let $LN(\Gamma)$ denote the minimum light number of Γ. Prof. Wu gave a characterization for $LN(\Gamma)$. In particular, if Γ is a line graph of a tree then $LN(\Gamma)$ is equal to the edge isoperimetric number of Γ.

$$max_k min_{\#S=k} \#EB(S).$$

The key of my approach is ...

1. We view each move for which we choose a vertex s as an invertible linear transformation, denoted by κ_s.
2. The lit-only σ-game on a simple graph can be considered as a representation of the simply-laced Coxeter group associated with Γ.
3. The transpose of κ_s preserves a symplectic form.

Conclusion

	nondeg. graphs	deg. graphs
line graphs	$\surd$	$\surd$
non-line graphs	$\surd$	?

Thanks for your attention